1. Convert the following complex numbers from rectangular form to polar form.

   (a) $3 + j4$
   Note that this complex number is in the 1st quadrant. The magnitude is $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$. The absolute value of the angle with respect to the $+x$-axis is $\tan^{-1}(4/3) = 53.13°$. Therefore, the polar representation is $5\angle 53.13°$, which is a shorthand for $5e^{j53.13°}$.

   (b) $-5 + j10$
   Note that this complex number is in the 2nd quadrant. The magnitude is $\sqrt{5^2 + 10^2} = \sqrt{125} = 11.18$. The absolute value of the angle measured from the $-x$-axis is $\tan^{-1}(10/5) = 63.43°$; therefore, the angle as measured from the $+x$-axis is $180° - 63.43° = 116.57°$. As a result, the polar representation is $11.18\angle 116.57°$, which is a shorthand for $11.18e^{j116.57°}$.

   (c) $-10 - j10$
   Note that this complex number is in the 3rd quadrant. The magnitude is $\sqrt{10^2 + 10^2} = \sqrt{200} = 14.14$. The absolute value of the angle measured from the $-x$-axis is $\tan^{-1}(10/10) = 45°$; therefore, the angle as measured from the $+x$-axis is $180° + 45° = 225°$. As a result, the polar representation is $14.14\angle 225°$, which is a shorthand for $14.14e^{j225°}$. We could use $-135°$ as well.

   (d) $3 - j4$
   Note that this complex number is in the 4th quadrant. The magnitude is $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$. The absolute value of the angle as measured from the $+x$-axis is $\tan^{-1}(4/3) = 53.13°$. As a result, the polar representation is $5\angle - 53.13°$, which is a shorthand for $5e^{-j53.13°}$.

2. Simplify the following expressions, then convert the result to polar form.

   (a) $(-3 + j4)(-10 - j10)$
   Expanding the two factors (using the FOIL method) we get, $30 + j30 - j40 - j^2 40 = 30 + j30 - j40 - (-1)40 = 70 - j10$. Note that this complex number is in the 4th quadrant. The magnitude is $\sqrt{70^2 + 10^2} = \sqrt{5000} = 70.71$. The absolute value of the angle measured from the $-x$-axis is $\tan^{-1}(10/70) = 8.13°$; therefore, the angle as measured from the $+x$-axis is $-8.13°$. As a result, the polar representation is $70.71\angle - 8.13°$.
   Another method is to convert each of the factors in the product to polar form; therefore, we get $(-3 + j4)(-10 - j10) = (5e^{j126.87°})(14.14e^{j225°}) = 70.7e^{j351.87°} = 70.7e^{-j8.13°}$.

   (b) $\dfrac{10 + j10}{-5 - j5}$
   We can simplify this quotient by multiplying the top and the bottom by the complex conjugate of the denominator $(-5 + j5)$. We can then proceed by applying the FOIL method to the numerator and denominator as shown below.

$$\frac{10 + j10}{-5 - j5}\left(\frac{-5 + j5}{-5 + j5}\right) = \frac{-50 + j50 - j50 + j^2 50}{25 - j25 + j25 - j^2 25} = \frac{-100}{50} = -2 + j0 = 2\angle 180° = 2\angle - 180°$$

   Another method is to convert each of the factors in the quotient to polar form first. We get $(10 + j10)/(-5 - j5) = (14.14e^{j45°})/(7.07e^{j225°}) = (14.14/7.07)e^{j45°}\,e^{-j225°} = 2e^{-j180°}$.

3. Convert the following complex numbers from polar form to rectangular form.

   (a) $10\,e^{j\frac{\pi}{4}}$
   $10\,e^{j\frac{\pi}{4}} = 10\cos(\frac{\pi}{4}) + j10\sin(\frac{\pi}{4}) = 10(\sqrt{2}/2) + j10(\sqrt{2}/2) = 5\sqrt{2} + j5\sqrt{2}$.

   (b) $-2\,e^{j60°}$
   $-2\,e^{j60°} = -2\cos(60°) - j2\sin(60°) = -2(1/2) - j2(\sqrt{3}/2) = -1 - j\sqrt{3}$.

   (c) $-5\,e^{-j\frac{\pi}{3}}$
   $-5\,e^{-j\frac{\pi}{3}} = -5\cos(-\frac{\pi}{3}) - j5\sin(-\frac{\pi}{3})$. Recall that $\cos(-x) = \cos(x)$ and $\sin(-x) = -\sin(x)$. Therefore, we have $-5\cos(\frac{\pi}{3}) + j5\sin(\frac{\pi}{3}) = -\frac{5}{2} + j\frac{5\sqrt{3}}{2}$.

   (d) $10\,e^{j225°}$
   $10\,e^{j225°} = 10\cos(225°) + j10\sin(225°)$. Therefore, $-10\cos(45°) - j10\sin(45°) = -10(\sqrt{(2)}/2) - j10(\sqrt{(2)}/2) = -5\sqrt{2} - j5\sqrt{2}$.

4. Simplify the following expressions, then convert the result to rectangular form.

   (a) $(20\,e^{j\frac{\pi}{6}})(-4\,e^{j60^\circ})$
   $(20\,e^{j\frac{\pi}{6}})(-4\,e^{j60^\circ}) = -80\,e^{j30^\circ}\,e^{j60^\circ} = -80\,e^{j90^\circ} = -j80.$

   (b) $\dfrac{-2\,e^{-j\frac{3\pi}{2}}}{4\,e^{j225^\circ}}$
   $\dfrac{-2\,e^{-j\frac{3\pi}{2}}}{4\,e^{j225^\circ}} = (-2/4)(e^{-j270^\circ})/(e^{j225^\circ}) = (-2/4)(e^{-j270^\circ})(e^{-j225^\circ}) = (-1/2)(e^{-j495^\circ})$. This last result is equivalent to $(-1/2)(e^{-j135^\circ}) = (-1/2)(e^{j225^\circ})$. Our result can now be written as $-(1/2)\,[-\cos(45^\circ) - j\sin(45^\circ)] = (1/2)(\sqrt{2}/2) + j(1/2)(\sqrt{2}/2) = 1/(2\sqrt{2}) + j1/(2\sqrt{2})$. We could have also replaced -1 in our initial step with e^{j180° or e^{-j180° and proceeded as well.

5. Find the total complex impedance (in Ω) for the following circuit elements or combinations at an angular frequency $\omega = 2000\,\text{rad/s}$.

   (a) 100Ω resistor
   $Z = R = 100\,\Omega$

   (b) 10mH inductor
   $Z = j\omega L = j(2000)(10\times10^{-3}) = j20\,\Omega$

   (c) 100μF capacitor
   $Z = 1/(j\omega C) = 1/(j(2000)(100\times10^{-6})) = 1/(j0.2) = -j5\,\Omega$

   (d) 200Ω resistor in series with a 5mH inductor
   $Z = R + j\omega L = 200 + j(2000)(5\times10^{-3}) = (200 + j10)\,\Omega$

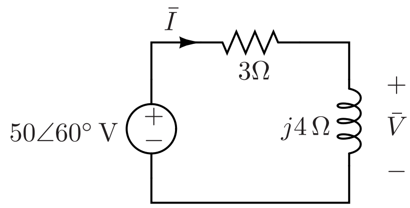
   (e) 500Ω resistor in series with a 50μF capacitor
   $Z = R + 1/(j\omega C) = 500 + 1/(j(2000)(50\times10^{-6})) = 500 + 1/(j0.1) = (500 - j10)\,\Omega$

   (f) $1\text{k}\Omega$ resistor in series with a 2mH inductor in series with a 20μF capacitor
   $Z = R + j\omega L + 1/(j\omega C) = 1000 + j(2000)(2\times10^{-3}) + 1/(j(2000)(20\times10^{-6})) = 500 + j4 + 1/(j0.04) = 1000 + j4 - j25 = (1000 - j21)\,\Omega$

   (g) 200Ω resistor in parallel with a 5mH inductor
   $Z = R||(j\omega L) = \dfrac{j\omega LR}{R + j\omega L} = \dfrac{j(2000)(5\times10^{-3})(200)}{200 + j(2000)(5\times10^{-3})} = \dfrac{j2000}{200 + j10}$. Now multiplying the top and the bottom by the complex conjugate of the denominator, we get
   $Z = \dfrac{j2000}{200 + j10}\left(\dfrac{200 - j10}{200 - j10}\right) = \dfrac{j400{,}000 - j^2 20{,}000}{40{,}000 + j2{,}000 - j2{,}000 - j^2 100}$
   $Z = \dfrac{20{,}000 + j400{,}000}{40{,}100} = \dfrac{20{,}000}{40{,}100} + j\dfrac{400{,}000}{40{,}100} = (0.4988 + j8.9751)\,\Omega$

   (h) 10mH inductor in parallel with a 100μF capacitor
   $Z = (j\omega L)||[1/(j\omega C)] = \dfrac{(j\omega L)[1/(j\omega C)]}{(j\omega L) + [1/(j\omega C)]} = \dfrac{j\omega L}{1 - \omega^2 LC} = \dfrac{j(2000)(10\times10^{-3})}{1 - (2000)^2(10\times10^{-3})(100\times10^{-6})}$
   Simplifying, we get $\dfrac{j20}{1-4} = -j\dfrac{20}{3} \approx -j6.67\,\Omega.$

6. Given the following circuit with complex impedances, find the complex current $\bar{I}$ and the complex voltage $\bar{V}$.



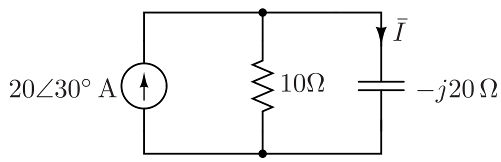
Since the $3\,\Omega$ resistor and the $j4\,\Omega$ are in series, the total impedance is $(3 + j4)\,\Omega$. The current $\bar{I}$ is simply the voltage source divided by the impedance.

$$\bar{I} = \frac{50\angle 60°}{Z_{total}} = \frac{50\angle 60°}{3 + j4} = \frac{50\angle 60°}{5\angle 53.1°} = 10\angle 6.9°$$

Note that the same current $\bar{I}$ flows through the resistor and the inductor (since they are in series). Therefore, the voltage across the inductor (marked as $\bar{V}$) is given by Ohm's Law.

$$\bar{V} = Z_{inductor}\bar{I} = (j4)(10\angle 6.9°) = (4\angle 90°)(10\angle 6.9°) = 40\angle 96.9°$$

7. Find the complex current $\bar{I}$ using current division.



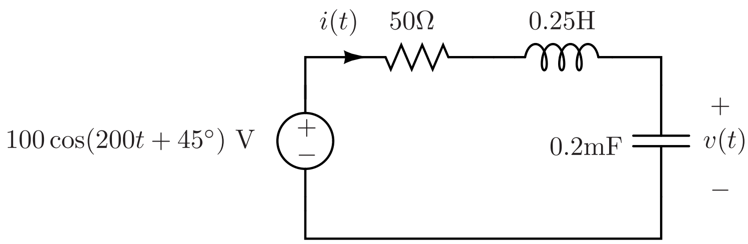
First denote the current source as $\bar{I}_s$ and the resistor and the capacitor as impedances $Z_1 = 10\,\Omega$ and $Z_2 = -j20\,\Omega$, respectively. Based on this, we can apply current division as we had done for resistors.

$$\bar{I} = \frac{Z_1}{Z_1 + Z_2} 20\angle 30° = \frac{10}{10 - j20}\, 20\angle 30° = \frac{(10\angle 0°)(20\angle 30°)}{22.36\angle -63.4°} = 8.94\angle 93.4°$$

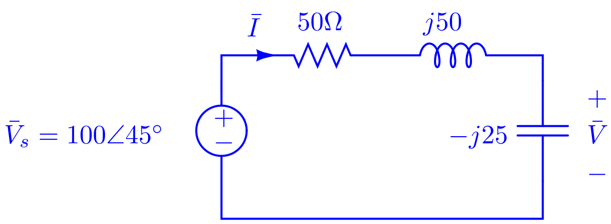
We are not asked to provide the rectangular form of the expression but if it was required we have to note that 93.4° is in the 2nd quadrant; therefore, the cosine of this angle will be negative while the sine will be positive. As a result, we get

$$\bar{I} = 8.94\angle 93.4° = 8.94\cos(93.4°) + j8.94\sin(93.4°) = -0.53 + j8.92$$

8. Find the complex current $\bar{I}$ and complex voltage $\bar{V}$, then find the corresponding time-domain current $i(t)$ and voltage $v(t)$. *Hint:* Start by converting the circuit shown below to its phasor equivalent to determine $\bar{I}$ and $\bar{V}$.



First draw the phasor equivalent circuit corresponding the problem using an angular frequency of $\omega = 200\text{rad/s}$. In the equivalent circuit, the voltage source becomes $\bar{V}_s = 100\angle 45°$. The impedances for the resistor, inductor and capacitor are 50, $j50$ and $-j25$ (all in Ω), respectively. The complete equivalent circuit is shown below, where the current and the voltage are denoted by their complex counterparts, $\bar{I}$ and $\bar{V}$, respectively.



We note from the circuit that the three impedances are in series (they share the same current $\bar{I}$); therefore, we can add the impedances to get the total impedance, $Z_{total} = 50 + j50 - j25 = 50 + j25$. Based on this, we can determine the complex current $\bar{I}$ similar to the approach in problem 6.

$$\bar{I} = \frac{\bar{V}_s}{Z_{total}} = \frac{100\angle 45°}{50 + j25} = \frac{100\angle 45°}{55.9\angle 26.6°} = 1.79\angle 18.4°$$

Since $\bar{I}$ represents the magnitude and phase of the current time function $i(t)$, we can write it as

$$\bar{I} = 1.79\angle 18.4° \Rightarrow i(t) = 1.79\cos(200t + 18.4°)\text{ A}$$

Getting back to the phasor circuit, we see that voltage $\bar{V}$ across the capacitor can be given by Ohm's Law as $\bar{V} = Z_c\bar{I}$, where $Z_c = -j25$ is the impedance of the capacitor.

$$\bar{V} = (-j25)\bar{I} = (-j25)(1.79\angle 18.4°) = (25\angle -90°)(1.79\angle 18.4°) = 44.75\angle -71.6°$$

Since $\bar{V}$ represents the magnitude and phase of the voltage time function $v(t)$, we can write it as

$$\bar{V} = 44.75\angle -71.6° \Rightarrow v(t) = 44.75\cos(200t - 71.6°)\text{ V}$$